Jagua Publication

Computational Methods, 3(2):18–26, July-December: 2026.
DOI: https://doi.org/10.58614/cm323

Computational Methods
ISSN (Online): 3048-7447

Journal homepage: https://jpub.org/journal-details.php?journal-id=43

Research Article

Complete Lattice Representation of Fuzzy Soft Topology through Matrices

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Article Info

Keywords: *Fuzzy soft sets, Matrix algebra, Partial order, Regular open sets, Regular closed sets.*

Received: *06.08.2026;*
Accepted: *01.09.2026;*
Published: *07.09.2026*

Abstract

This paper develops a matrix-theoretic framework for the study of fuzzy soft topological spaces. A correspondence between fuzzy soft topologies and collections of fuzzy soft matrices is established, demonstrating that a fuzzy soft topology is equivalent to a complete distributive lattice of fuzzy soft matrices. This correspondence enables topological properties to be investigated through algebraic matrix operations. An order relation on fuzzy soft matrices is introduced and its fundamental properties are examined. Matrix-valued lower and upper operators are then defined to characterize fuzzy soft interior and fuzzy soft closure in an algebraic setting. The developed operators provide concise matrix representations for interior, closure, regular open, regular closed, semi-open, pre-open, α-open, β-open, b-open, $*b$-open, $b^{\#}$-open and their corresponding closed classes. Several equivalence theorems are established, showing that these generalized fuzzy soft topological sets can be completely characterized by matrix inequalities and lattice identities. The proposed approach unifies topological and algebraic viewpoints, simplifies the representation of fuzzy soft topological concepts, and provides a systematic foundation for further investigations in fuzzy soft topology, lattice theory, and matrix-based computational methods.

1. Introduction

Fuzzy soft set theory provides an effective mathematical framework for representing uncertainty that arises from both fuzziness and parameter dependence [1]. Owing to its flexibility, fuzzy soft topology has become an important research area for studying generalized notions of openness, closure, continuity, and related topological structures [2, 3]. Although many generalized fuzzy soft open and closed sets have been introduced, their investigation is generally carried out through set-theoretic operations, making the algebraic structure of these classes less explicit.

Matrix representations offer a natural and compact way of expressing fuzzy soft sets since every fuzzy soft set can be represented by a matrix of membership values [4]. Matrix operations also provide computational advantages and allow topological concepts to be interpreted through algebraic structures. Establishing a rigorous correspondence between fuzzy soft topology and matrix theory therefore creates new opportunities for analyzing fuzzy soft spaces using lattice-theoretic methods.

In this work, a matrix formulation of fuzzy soft topology is developed. It is shown that a fuzzy soft topology is equivalent to a complete distributive lattice of fuzzy soft matrices, thereby establishing an algebraic characterization of fuzzy soft topological spaces. A partial order on fuzzy soft matrices is introduced and several of its basic properties are investigated. Based on this ordering, lower and upper matrix operators are defined, which serve as algebraic analogues of fuzzy soft interior and fuzzy soft closure operators.

How To Cite: P.R. Devi, and M. Kiruthika. Complete Lattice Representation of Fuzzy Soft Topology through Matrices. *Computational Methods*, 3(2):18–26, 2026. https://doi.org/10.58614/cm323

The proposed operators are further employed to characterize a wide range of generalized fuzzy soft topological sets. Matrix-based criteria are obtained for fuzzy soft open, closed, regular open, regular closed, semi-open, pre-open, α-open, β-open, b-open, $*b$-open, $b^{\#}$-open, and their corresponding closed classes. Each characterization is expressed through equivalent matrix identities or order relations, providing a unified and computationally convenient description of these generalized classes.

2. Preliminaries

2.1. Soft sets

Throughout this paper, X is a non-empty set and E is a parameter space. If $A \subseteq E$ and $F : A \to 2^X$, then the pair (F,A) is called a soft subset of X over the parameter space. Let $\widetilde{S}(X,E)$ denote the collection of all soft subsets of X with parameter space E [5, 6].
The pair (F,A) can be identified with $(F,E) = \widetilde{F}$ by defining

$$\widetilde{F}(e) = \varnothing \quad \text{for all } e \in E \setminus A, \qquad \widetilde{F}(e) = F(e) \subseteq X.$$

Let $\widetilde{\varnothing}$ and $\widetilde{X}$ denote the null soft subset and absolute soft subset of X, defined by

$$\widetilde{\varnothing}(e) = \varnothing, \qquad \widetilde{X}(e) = X, \qquad e \in E.$$

For two soft sets $\widetilde{F}, \widetilde{G} \in \widetilde{S}(X,E)$, we say that

$$\widetilde{F} \sqsubseteq \widetilde{G} \quad \Longleftrightarrow \quad \widetilde{F}(e) \subseteq \widetilde{G}(e), \quad e \in E.$$

The operations $\widetilde{F} \sqcap \widetilde{G}$, $\widetilde{F} \sqcup \widetilde{G}$, $\widetilde{F} \setminus \widetilde{G}$, and $\widetilde{X} \setminus \widetilde{G}$ denote soft intersection, soft union, soft difference, and soft complement, respectively.

Definition 2.1. *For every $e \in E$,*

$$(\widetilde{F} \sqcap \widetilde{G})(e) = F(e) \cap G(e),$$
$$(\widetilde{F} \sqcup \widetilde{G})(e) = F(e) \cup G(e),$$
$$(\widetilde{F} \setminus \widetilde{G})(e) = F(e) \setminus G(e),$$
$$(\widetilde{X} \setminus \widetilde{G})(e) = X \setminus G(e).$$

2.2. Soft topology

Definition 2.2. *Let $\widetilde{\tau}$ be a subcollection of $\widetilde{S}(X,E)$ [7–9]. Then $\widetilde{\tau}$ is said to be a soft topology on X if:*

(i) $\widetilde{\varnothing}, \widetilde{X} \in \widetilde{\tau}$;
(ii) $\widetilde{\tau}$ is closed under finite soft intersection;
(iii) $\widetilde{\tau}$ is closed under arbitrary soft union.

If $\widetilde{\tau}$ is a soft topology on X, then $(X, \widetilde{\tau}, E)$ is called a soft topological space. The members of $\widetilde{\tau}$ are called soft open sets.

2.3. Fuzzy sets

Definition 2.3. *If X is a universe of discourse [10–12], then a function*

$$\mu : X \longrightarrow [0,1]$$

is called a fuzzy subset of X and is represented by

$$\{(x, \mu(x)) : x \in X\}.$$

The value $\mu(x)$ is called the membership grade of x.

If μ assumes only the values 0 and 1, then it is called a crisp subset of X.

2.4. Fuzzy soft set

Let X be an initial universal set and let E be a set of parameters. Let $[0,1]^X$ denote the collection of all fuzzy subsets of X [13]. If $A \subseteq E$ and

$$\widetilde{\mu} : A \longrightarrow [0,1]^X,$$

then $(\widetilde{\mu}, A)$ is known as a fuzzy soft set in X with E as the parameter space.
If $(\widetilde{\mu}, A)$ is a fuzzy soft set, then for every parameter $e \in A$, $\widetilde{\mu}(e)$ is a fuzzy subset of X [14, 15]. If we set $\widetilde{\mu}(e) = 0^*$ for every $e \in E \setminus A$, then $\widetilde{\mu} : E \to [0,1]^X$ is a fuzzy soft set over (X,E). Let $\widetilde{FS}(X,E)$ denote the collection of all fuzzy soft sets over (X,E).
Let $\widetilde{0}$ and $\widetilde{1}$ denote the null and absolute fuzzy soft subsets:

$$\widetilde{0}(e) = 0^*, \qquad \widetilde{1}(e) = 1^*, \qquad e \in E.$$

For $\widetilde{\mu}_1, \widetilde{\mu}_2 \in \widetilde{FS}(X,E)$, we write

$$\widetilde{\mu}_1 \preceq \widetilde{\mu}_2$$

if $\widetilde{\mu}_1(e) \leq \widetilde{\mu}_2(e)$ for every $e \in E$.

Definition 2.4. *For every $e \in E$,*

$$(\widetilde{\mu}_1 \sqcap \widetilde{\mu}_2)(e) = \widetilde{\mu}_1(e) \wedge \widetilde{\mu}_2(e),$$
$$(\widetilde{\mu}_1 \sqcup \widetilde{\mu}_2)(e) = \widetilde{\mu}_1(e) \vee \widetilde{\mu}_2(e),$$
$$(\widetilde{1} \setminus \widetilde{\mu}_2)(e) = \big(\widetilde{\mu}_2(e)\big)'.$$

2.5. Fuzzy soft topology

Definition 2.5. *Let $\widetilde{\tau}$ be a subcollection of $\widetilde{FS}(X,E)$. Then $\widetilde{\tau}$ is said to be a fuzzy soft topology on X if:*

(i) $\widetilde{0}, \widetilde{1} \in \widetilde{\tau}$;
(ii) $\widetilde{\tau}$ *is closed under finite fuzzy soft intersection;*
(iii) $\widetilde{\tau}$ *is closed under arbitrary fuzzy soft union.*

If $\widetilde{\tau}$ is a fuzzy soft topology on X, then $(X,\widetilde{\tau},E)$ is called a fuzzy soft topological space.

If $\widetilde{\mu}$ is a fuzzy soft subset of $(X,\widetilde{\tau},E)$, then $\mathrm{fsInt}\,\widetilde{\mu}$ and $\mathrm{fsCl}\,\widetilde{\mu}$ denote the fuzzy soft interior and fuzzy soft closure.

3. Fuzzy soft topology versus matrices

Let $\widetilde{\tau}$ be a collection of fuzzy soft sets in $\widetilde{FS}(X,E)$ and define

$$FM(\widetilde{\tau}) = \{\mathrm{Mat}(\widetilde{\mu}) : \widetilde{\mu} \in \widetilde{\tau}\}.$$

Theorem 3.1. *The following are equivalent:*

(i) $\widetilde{\tau}$ *is a fuzzy soft topology.*
(ii) $(FM(\widetilde{\tau}), \oplus, \odot, [0_{ij}], [1_{ij}])$ *is a complete distributive lattice.*
(iii) $(\widetilde{\tau}, \oplus, \odot, \widetilde{0}, \widetilde{1})$ *is a complete distributive lattice.*

Proof. Suppose $\widetilde{\tau}$ is a fuzzy soft topology. Then

$$\mathrm{Mat}(\widetilde{0}) = [0_{ij}] \in FM(\widetilde{\tau}), \qquad \mathrm{Mat}(\widetilde{1}) = [1_{ij}] \in FM(\widetilde{\tau}).$$

For $\widetilde{\mu}_1, \widetilde{\mu}_2 \in \widetilde{\tau}$, closure under fuzzy soft union and intersection gives

$$\widetilde{\mu}_1 \sqcap \widetilde{\mu}_2, \quad \widetilde{\mu}_1 \sqcup \widetilde{\mu}_2 \in \widetilde{\tau}.$$

Under the corresponding matrix operations,

$$\mathrm{Mat}(\widetilde{\mu}_1) \odot \mathrm{Mat}(\widetilde{\mu}_2) = \mathrm{Mat}(\widetilde{\mu}_1 \sqcap \widetilde{\mu}_2),$$

and

$$\mathrm{Mat}(\widetilde{\mu}_1) \oplus \mathrm{Mat}(\widetilde{\mu}_2) = \mathrm{Mat}(\widetilde{\mu}_1 \sqcup \widetilde{\mu}_2).$$

Hence $FM(\widetilde{\tau})$ forms a complete distributive lattice. The converse follows by reversing these implications. □

Theorem 3.2. *A fuzzy soft set $\widetilde{\mu}$ is fuzzy soft closed if and only if*

$$\mathrm{Mat}(\widetilde{\mu}') \in FM(\widetilde{\tau}).$$

Proof. We have

$$\widetilde{\mu} \text{ is fuzzy soft closed} \Longleftrightarrow \widetilde{1} \setminus \widetilde{\mu} \in \widetilde{\tau} \Longleftrightarrow \mathrm{Mat}(\widetilde{1} \setminus \widetilde{\mu}) \in FM(\widetilde{\tau}),$$

and therefore

$$\mathrm{Mat}(\widetilde{\mu}') \in FM(\widetilde{\tau}).$$

□

Definition 3.3. *For fuzzy soft matrices, define*

$$\mathrm{Mat}(\widetilde{\mu}_1) \leq \mathrm{Mat}(\widetilde{\mu}_2)$$

if

$$\mathrm{Mat}(\widetilde{\mu}_1) \oplus \mathrm{Mat}(\widetilde{\mu}_2) = \mathrm{Mat}(\widetilde{\mu}_2).$$

Theorem 3.4. *The following are equivalent:*

(i) $\mathrm{Mat}(\widetilde{\mu}_1) \leq \mathrm{Mat}(\widetilde{\mu}_2)$;
(ii) $\mathrm{Mat}(\widetilde{\mu}_1 \oplus \widetilde{\mu}_2) = \mathrm{Mat}(\widetilde{\mu}_2)$;
(iii) $\widetilde{\mu}_1 \oplus \widetilde{\mu}_2 = \widetilde{\mu}_2$;

(iv) $\widetilde{\mu}_1 \sqcup \widetilde{\mu}_2 = \widetilde{\mu}_2$;
(v) $\widetilde{\mu}_1 \preceq \widetilde{\mu}_2$;
(vi) $\widetilde{\mu}_1 \sqcap \widetilde{\mu}_2 = \widetilde{\mu}_1$;
(vii) $\mathrm{Mat}(\widetilde{\mu}_1) \odot \mathrm{Mat}(\widetilde{\mu}_2) = \mathrm{Mat}(\widetilde{\mu}_1)$;
(viii) $\widetilde{\mu}_1 \odot \widetilde{\mu}_2 = \widetilde{\mu}_1$.

Proof. The equivalences follow from the definitions of the matrix operations and the pointwise maximum and minimum operations:

$$\max\{(\mu_1)_{ij}, (\mu_2)_{ij}\} = (\mu_2)_{ij}$$

and

$$\min\{(\mu_1)_{ij}, (\mu_2)_{ij}\} = (\mu_1)_{ij}.$$

Thus all the stated conditions are equivalent. □

Proposition 3.5. *The relation $\leq$ is a partially ordered relation on $FM(\widetilde{FS}(X,E))$, but it is not linearly ordered.*

Proposition 3.6.

$$\mathrm{Mat}(\widetilde{\mu}_1 \odot \widetilde{\mu}_2) \leq \mathrm{Mat}(\widetilde{\mu}_1 \oplus \widetilde{\mu}_2).$$

Definition 3.7. *For $\mathrm{Mat}(\widetilde{\mu}_1)$ define the lower and upper matrix operators by*

$$\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \oplus\{\mathrm{Mat}(\widetilde{\mu}) : \mathrm{Mat}(\widetilde{\mu}) \leq \mathrm{Mat}(\widetilde{\mu}_1),\ \widetilde{\mu} \in \widetilde{\tau}\}, \tag{1}$$

$$\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \odot\{\mathrm{Mat}(\widetilde{\mu}) : \mathrm{Mat}(\widetilde{\mu}_1) \leq \mathrm{Mat}(\widetilde{\mu}),\ \widetilde{\mu} \in \widetilde{1} \setminus \widetilde{\tau}\}. \tag{2}$$

Furthermore,

$$\widetilde{\mathfrak{L}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \widetilde{\mathfrak{L}}(\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1))),$$

$$\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \widetilde{\mathfrak{L}}(\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1))),$$

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \widetilde{\mathfrak{U}}(\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1))),$$

and

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \widetilde{\mathfrak{U}}(\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1))).$$

Theorem 3.8. *For every fuzzy soft matrix,*

(i)

$$\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) \leq \mathrm{Mat}(\widetilde{\mu}_1) \leq \widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1));$$

(ii) if $\mathrm{Mat}(\widetilde{\mu}_1) \leq \mathrm{Mat}(\widetilde{\mu}_2)$, *then*

$$\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) \leq \widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_2))$$

and

$$\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)) \leq \widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_2));$$

(iii)

$$\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \mathrm{Mat}(\widetilde{\mu}_1) \Longleftrightarrow \widetilde{\mu}_1 \in \widetilde{\tau};$$

(iv)

$$\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \mathrm{Mat}(\widetilde{\mu}_1) \Longleftrightarrow \widetilde{\mu}_1 \in \widetilde{1} \setminus \widetilde{\tau};$$

(v)

$$\widetilde{\mathfrak{L}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1));$$

(vi)

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)) = \widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)).$$

Theorem 3.9.

$$\begin{aligned}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) &\leq \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) \leq \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)) \\ &\leq \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)) \leq \widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)).\end{aligned}$$

Also,

$$\begin{aligned}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) &\leq \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) \leq \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}_1)) \\ &\leq \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)) \leq \widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}_1)).\end{aligned}$$

4. Characterizations of fuzzy soft topological sets

Proposition 4.1. *For every fuzzy soft set $\widetilde{\mu}$,*

$$\mathrm{Mat}(\mathrm{fsInt}\,\widetilde{\mu}) = \widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}))$$

and

$$\mathrm{Mat}(\mathrm{fsCl}\,\widetilde{\mu}) = \widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})).$$

Theorem 4.2.

$$\begin{aligned}
\mathrm{Mat}(\mathrm{fsInt}(\mathrm{fsCl}\,\widetilde{\mu})) &= \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})),\\
\mathrm{Mat}(\mathrm{fsCl}(\mathrm{fsInt}\,\widetilde{\mu})) &= \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})),\\
\mathrm{Mat}(\mathrm{fsInt}(\mathrm{fsCl}(\mathrm{fsInt}\,\widetilde{\mu}))) &= \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})),\\
\mathrm{Mat}(\mathrm{fsCl}(\mathrm{fsInt}(\mathrm{fsCl}\,\widetilde{\mu}))) &= \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})).
\end{aligned}$$

Theorem 4.3. *The following characterizations hold:*

(i) $\widetilde{\mu}$ *is fuzzy soft open iff*

$$\mathrm{Mat}(\widetilde{\mu}) = \widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})).$$

(ii) $\widetilde{\mu}$ *is fuzzy soft closed iff*

$$\mathrm{Mat}(\widetilde{\mu}) = \widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})).$$

(iii) $\widetilde{\mu}$ *is fuzzy soft regular open iff*

$$\mathrm{Mat}(\widetilde{\mu}) = \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})).$$

(iv) $\widetilde{\mu}$ *is fuzzy soft regular closed iff*

$$\mathrm{Mat}(\widetilde{\mu}) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})).$$

(v) $\widetilde{\mu}$ *is a fuzzy Q-soft set iff*

$$\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})).$$

(vi) $\widetilde{\mu}$ *is fuzzy soft $b^{\#}$-open iff*

$$\mathrm{Mat}(\widetilde{\mu}) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})).$$

(vii) $\widetilde{\mu}$ *is fuzzy soft $b^{\#}$-closed iff*

$$\mathrm{Mat}(\widetilde{\mu}) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})).$$

Theorem 4.4. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft semi-open;*

(ii)

$$\mathrm{Mat}(\widetilde{\mu}) \le \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \mathrm{Mat}(\widetilde{\mu}).$$

Theorem 4.5. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft pre-open;*

(ii)

$$\mathrm{Mat}(\widetilde{\mu}) \le \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \mathrm{Mat}(\widetilde{\mu}).$$

Theorem 4.6. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft* α*-open;*

(ii)

$$\mathrm{Mat}(\widetilde{\mu}) \leq \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \mathrm{Mat}(\widetilde{\mu}).$$

Theorem 4.7. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft* β*-open;*

(ii)

$$\mathrm{Mat}(\widetilde{\mu}) \leq \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \mathrm{Mat}(\widetilde{\mu}).$$

Theorem 4.8. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is a q-fuzzy soft set;*

(ii)

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \leq \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iii)

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iv)

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})).$$

Theorem 4.9. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is a p-fuzzy soft set;*

(ii)

$$\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) \leq \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iii)

$$\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) \oplus \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iv)

$$\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})).$$

Theorem 4.10. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft b-open;*

(ii)

$$\mathrm{Mat}(\widetilde{\mu}) \leq \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\left(\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\right);$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\left(\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\right) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\left(\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\right);$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\left(\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\right) = \mathrm{Mat}(\widetilde{\mu}).$$

Theorem 4.11. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft* $*$*b-open;*

(ii)

$$\mathrm{Mat}(\widetilde{\mu}) \leq \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \Big(\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\Big) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \Big(\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\Big) = \mathrm{Mat}(\widetilde{\mu}).$$

Theorem 4.12. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft semi-closed;*
(ii)

$$\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) \leq \mathrm{Mat}(\widetilde{\mu});$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \mathrm{Mat}(\widetilde{\mu});$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})).$$

Theorem 4.13. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft pre-closed;*
(ii)

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \leq \mathrm{Mat}(\widetilde{\mu});$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \mathrm{Mat}(\widetilde{\mu});$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})).$$

Theorem 4.14. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft* α*-closed;*
(ii)

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) \leq \mathrm{Mat}(\widetilde{\mu});$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \mathrm{Mat}(\widetilde{\mu});$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})).$$

Theorem 4.15. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft* β*-closed;*
(ii)

$$\widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \leq \mathrm{Mat}(\widetilde{\mu});$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \mathrm{Mat}(\widetilde{\mu});$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) = \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})).$$

Theorem 4.16. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft b-closed;*
(ii)

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) \leq \mathrm{Mat}(\widetilde{\mu});$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \left(\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\right) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \left(\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \odot \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\right) = \mathrm{Mat}(\widetilde{\mu}).$$

Theorem 4.17. *The following are equivalent:*

(i) $\widetilde{\mu}$ *is fuzzy soft* $*b$*-closed;*

(ii)

$$\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu})) \leq \mathrm{Mat}(\widetilde{\mu});$$

(iii)

$$\mathrm{Mat}(\widetilde{\mu}) \odot \left(\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\right) = \widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}));$$

(iv)

$$\mathrm{Mat}(\widetilde{\mu}) \oplus \left(\widetilde{\mathfrak{U}}\widetilde{\mathfrak{L}}(\mathrm{Mat}(\widetilde{\mu})) \oplus \widetilde{\mathfrak{L}}\widetilde{\mathfrak{U}}(\mathrm{Mat}(\widetilde{\mu}))\right) = \mathrm{Mat}(\widetilde{\mu}).$$

5. Conclusion

A comprehensive matrix framework for fuzzy soft topology has been established by representing fuzzy soft topological structures through fuzzy soft matrices and lattice operations. The equivalence between fuzzy soft topologies and complete distributive lattices of fuzzy soft matrices provides a unified algebraic foundation for studying fuzzy soft spaces.

The introduced partial order and the associated lower and upper matrix operators successfully characterize fuzzy soft interior and closure in an entirely matrix-based setting. Using these operators, numerous generalized fuzzy soft topological classes, including regular open, regular closed, semi-open, pre-open, α-open, β-open, b-open, $*b$-open, $b^{\#}$-open, and their corresponding closed counterparts, are characterized by equivalent matrix equations and order relations.

These characterizations demonstrate that diverse fuzzy soft topological properties can be interpreted through a common lattice-theoretic framework, leading to a simpler and more systematic treatment of generalized fuzzy soft sets. The proposed methodology strengthens the relationship between fuzzy soft topology and matrix algebra while providing an efficient representation for theoretical analysis. The developed framework offers a useful basis for extending matrix methods to other generalized fuzzy soft topological structures and for developing computational algorithms that support applications involving uncertainty, parameterized information systems, and intelligent decision-making.

Article Information

Funding: The authors received no external funding.

Author Contributions: P. Rohini Devi: Conceptualization, Methodology, Formal analysis, Writing – original draft; M. Kiruthika: Methodology, Writing – review & editing, Supervision.

Conflict of Interest: The authors declare no conflict of interest.

Disclaimer (Artificial Intelligence): The author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.), and text-to-image generators have been used during writing or editing of manuscripts.

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