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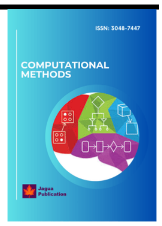

Research Article

The equation of order 6 of the NLS hierarchy

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Abstract

The equation of order 6 of the NLS hierarchy is considered and particular solutions are given. Explicit expressions of these solutions are constructed for the first orders depending on multi-parameters. We study patterns of these solutions in the (x,t) plane according the different values of the parameters. So series of rogue waves are highlighted.

PACS numbers: 33Q55, 37K10

1. Introduction

We are interested here in the hierarchy of the NLS equation, in particular its order 6 equation which can be written in the form

$$\begin{array}{l} u_t+u_{7x}+14|u^2|u_{5x}+42\overline{u}u_xu_{4x}+28u\overline{u}_xu_{4x}+14uu_x\overline{u}_{4x}+70|u|^4u_{3x} \\ +98|u_x|^2u_{3x}+70\overline{u}u_{2x}u_{3x}+42u\overline{u}_{2x}u_{3x}+28u_x^2\overline{u}_{3x}+28uu_{2x}\overline{u}_{3x}+280\overline{u}|u|^2u_xu_{2x} \\ +140|u|^2u\overline{u}_xu_{2x}+140|u|^2uu_x\overline{u}_{2x}+70\overline{u}u_{2x}^2+112u_x|u_{2x}|^2+70\overline{u}^2u_x^3+280|u|^2|u_x|^2u_x \\ +70u^2|u_x|^2\overline{u}_x+140|u|^6u_x \end{array} \quad (1)$$

the subscripts meaning partial derivatives and $\overline{u}$ the complex conjugate of u.
The first equation of this NLS hierarchy is the well known NLS equation [1].
The second one is the mKdV equation [2].
The third one is the LPD equation [3].

A lot of works has been done on these three equations.
One can mention, for the first equation [4], for the second one [5] and the third one [3, 6, 7].
But, for the following orders of this hierarchy, very few studies have been carried out. We construct here explicit solutions of the equation of order 6 of this hierarchy for the first orders and study the patterns of the modulus of the solutions in the (x,t) plane.

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2. Solution of order 1 to the sixth equation of the NLS hierarchy

Theorem 2.1. *The function $v(x,t)$ defined by*

$$v(x,t) = \frac{-3+4x^2-1120xt+78400t^2}{1+4x^2-1120xt+78400t^2} \tag{2}$$

is a solution to the $(NLS6)$ equation (1).

Proof: It is sufficient to replace the expression of the solution given by (2) and check that (1) is satisfied.

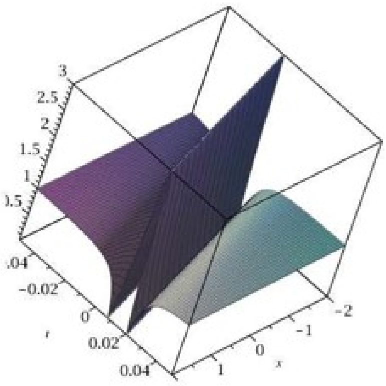

Figure 1: Solution of order 1 to (NLS6)

It is important to emphasize that we get a non smooth solution of the equation (1) as in the case of the mKdV equation [8]or the equation of order 4 of this NLS hierarchy [9].

3. Soltions of order 2 to the sixth equation of the NLS hierarchy

Theorem 3.1. *The function $v(x,t)$ defined by*

$$v(x,t) = \frac{n(x,t)}{d(x,t)} \tag{3}$$

with

$n(x,t) = -(-64x^6+53760tx^5+2304b_1x^5+144x^4-34560{b_1}^2x^4+768ia_1x^4-18816000t^2x^4-1612800b_1tx^4-768{a_1}^2x^4+430080t{a_1}^2x^3+19353600{b_1}^2tx^3+18432b_1{a_1}^2x^3+276480{b_1}^3x^3-188160tx^3-430080ia_1tx^3-18432ia_1x^3b_1-4992b_1x^3+3512320000t^3x^3+451584000b_1t^2x^3-368793600000t^4x^2-165888{b_1}^2{a_1}^2x^2+6144i{a_1}^3x^2+90316800ia_1t^2x^2+4032000b_1tx^2-7741440b_1t{a_1}^2x^2+5760{a_1}^2x^2+58752{b_1}^2x^2-4064256000{b_1}^2t^2x^2+7741440ia_1b_1tx^2-90316800t^2{a_1}^2x^2+62092800t^2x^2-1244160{b_1}^4x^2+180x^2-63221760000b_1t^3x^2-3072{a_1}^4x^2-116121600{b_1}^3tx^2-1152ia_1x^2+165888ia_1{b_1}^2x^2-322560t{a_1}^2x+4425523200000b_1t^4x+1083801600b_1t^2{a_1}^2x+379330560000{b_1}^2t^3x-835430400b_1t^2x+348364800{b_1}^4tx+16257024000{b_1}^3t^2x+46448640{b_1}^2t{a_1}^2x-4608ia_1xb_1-1083801600ia_1xb_1t^2-7902720000t^3x+2985984{b_1}^5x-28062720{b_1}^2tx+663552{b_1}^3{a_1}^2x-50688b_1{a_1}^2x-1720320i{a_1}^3tx+8429568000t^3{a_1}^2x-46448640ia_1x{b_1}^2t-73728i{a_1}^3xb_1+36864b_1{a_1}^4x-663552ia_1x{b_1}^3-5616b_1x-967680ia_1tx-290304{b_1}^3x+20652441600000t^5x+860160t{a_1}^4x-8429568000ia_1t^3x-292320tx-720ia_1+1536i{a_1}^3-50577408000b_1t^3{a_1}^2-5160960b_1t{a_1}^4+995328ia_1{b_1}^4+10321920i{a_1}^3b_1t-645120b_1t{a_1}^2-45+8386560ia_1b_1t+350353920000t^4+63866880{b_1}^3t+12288i{a_1}^5-92897280{b_1}^3t{a_1}^2+295034880000ia_1t^4+96768{b_1}^2{a_1}^2-67737600t^2{a_1}^2+51631104000b_1t^3+120422400i{a_1}^3t^2-7761600t^2+947520b_1t+221184i{a_1}^3{b_1}^2-481890304000000t^6-3251404800{b_1}^2t^2{a_1}^2-13276569600000{b_1}^2t^4+2777241600{b_1}^2t^2-24385536000{b_1}^4t^2-758661120000{b_1}^3t^3-995328{b_1}^4{a_1}^2-110592{b_1}^2{a_1}^4-123914649600000b_1t^5-295034880000t^4{a_1}^2-60211200t^2{a_1}^4+69120ia_1{b_1}^2+158054400ia_1t^2+92897280ia_1{b_1}^3t+3251404800ia_1{b_1}^2t^2+50577408000ia_1b_1t^3-418037760{b_1}^5t+8448{a_1}^4+518400{b_1}^4-2985984{b_1}^6-4096{a_1}^6+1872{a_1}^2+18000{b_1}^2)\mathrm{e}^{2ia_1}$

and

$d(x,t) = 64x^6-2304b_1x^5-53760tx^5+34560{b_1}^2x^4+1612800b_1tx^4+48x^4+18816000t^2x^4+768{a_1}^2x^4-430080t{a_1}^2x^3+384b_1x^3-19353600{b_1}^2tx^3-18432b_1{a_1}^2x^3-3512320000t^3x^3+80640tx^3-276480{b_1}^3x^3-451584000b_1t^2x^3-2096640b_1tx^2+368793600000t^4x^2-1152{a_1}^2x^2+108x^2+165888{b_1}^2{a_1}^2x^2+7741440b_1t{a_1}^2x^2+4064256000{b_1}^2t^2x^2+116121600{b_1}^3tx^2+63221760000b_1t^3x^2+90316800t^2{a_1}^2x^2-39513600t^2x^2+1244160{b_1}^4x^2-17280{b_1}^2x^2+3072{a_1}^4x^2-1083801600b_1t^2{a_1}^2x+16450560{b_1}^2tx-967680t{a_1}^2x-110880tx-379330560000{b_1}^2t^3x+5795328000t^3x+124416{b_1}^3x-348364800{b_1}^4tx-16257024000{b_1}^3t^2x-4608b_1{a_1}^2x-4425523200000b_1t^4x-2448b_1x-46448640{b_1}^2t{a_1}^2x-663552{b_1}^3{a_1}^2x-2985984{b_1}^5x-36864b_1{a_1}^4x-860160t{a_1}^4x+564480000b_1t^2x-20652441600000t^5x-8429568000t^3{a_1}^2x+5160960b_1t{a_1}^4-40642560{b_1}^3t+69120{b_1}^2{a_1}^2-1964390400{b_1}^2t^2-38986752000b_1t^3+58564800t^2+158054400t^2{a_1}^2+8386560b_1t{a_1}^2-276595200000t^4+13276569600000{b_1}^2t^4+418037760{b_1}^5t+9+758661120000{b_1}^3t^3+995328{b_1}^4{a_1}^2+110592{b_1}^2{a_1}^4+24385536000{b_1}^4t^2+295034880000t^4{a_1}^2+60211200t^2{a_1}^4+2116800b_1t+123914649600000b_1t^5+481890304000000t^6+92897280{b_1}^3t{a_1}^2+3251404800{b_1}^2t^2{a_1}^2+50577408000b_1t^3{a_1}^2+6912{a_1}^4-269568{b_1}^4+2985984{b_1}^6+4096{a_1}^6+1584{a_1}^2+20016{b_1}^2$

is a solution to the (NLS6) equation (1).

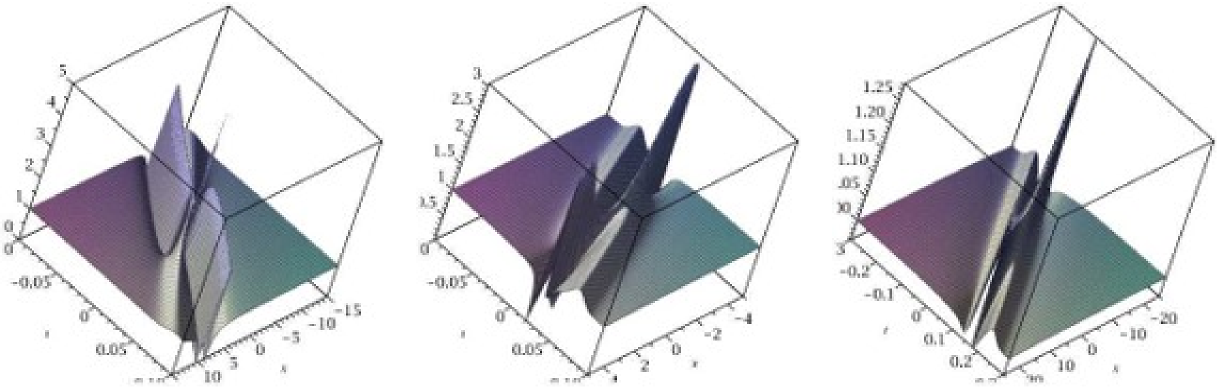

Figure 2: Solution of order 2 to the equation (1); to the left $a_1 = 0, b_1 = 0$; in the center $a_1 = 1, b_1 = 0$; to the right $a_1 = 3, b_1 = 0$

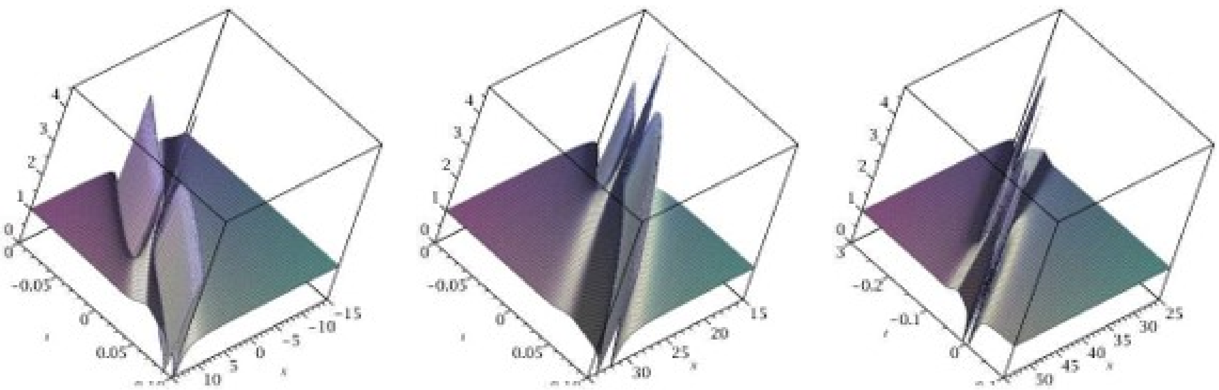

Figure 3: Solution of order 2 to the equation (1); to the left $a_1 = 0, b_1 = 1$; in the center $a_1 = 0, b_1 = 4$; to the right $a_1 = 0, b_1 = 10$

Proof: We have to replace the expression of the solution given by (3), and verify that the relation (1) is verified.

We get also a non smooth solution of the equation (1).
It is the same as in the case of the mKdV equation [8]or the equation of order 4 of this NLS hierarchy [9].

4. Solutions of order 3 to the sixth equation of the NLS hierarchy

The solution order 3 to the sixth equation of the NLS hierarchy is given by the following expression.

Theorem 4.1. *The function $v(x,t)$ defined by*

$$v(x,t) = \left(1 - 24\frac{n(x,t)}{d(x,t)}\right)\mathrm{e}^{i(2a_1-6a_2+20t)} \tag{4}$$

with
$n(x,t) = 675 - 86400(8b_1 - 80b_2 + 560t)(32b_2 - 448t) + 15(1 + (4a_1 - 24a_2)^2)(2x - 12b_1 + 60b_2 - 280t)^8 + (210 - 60(4a_1 - 24a_2)^2 +$
$50(4a_1 - 24a_2)^4 + 7680(4a_1 - 24a_2)a_2)(2x - 12b_1 + 60b_2 - 280t)^6 + (-720(4a_1 - 24a_2)^2(8b_1 - 80b_2 + 560t) - 5760b_1 - 11520b_2 +$
$564480t)(2x - 12b_1 + 60b_2 - 280t)^5 - 172800(4a_1 - 24a_2)^3 a_2 + (450(4a_1 - 24a_2)^2 - 150(4a_1 - 24a_2)^4 + 70(4a_1 - 24a_2)^6 + 19200(4a_1 -$
$24a_2)^3 a_2 - 450 + 5400(8b_1 - 80b_2 + 560t)^2 - 460800a_2^2 + 57600(4a_1 - 24a_2)a_2)(2x - 12b_1 + 60b_2 - 280t)^4 + (-2400(4a_1 - 24a_2)^4(8b_1 -$
$80b_2 + 560t) + 460800(4a_1 - 24a_2)(8b_1 - 80b_2 + 560t)a_2 + 57600b_1 - 806400b_2 + 7257600t - 7200(4a_1 - 24a_2)^2(16b_1 - 128b_2 +$
$672t))(2x - 12b_1 + 60b_2 - 280t)^3 + (6750(4a_1 - 24a_2)^4 + 420(4a_1 - 24a_2)^6 + 45(4a_1 - 24a_2)^8 - 2700(4a_1 - 24a_2)^2(5 + 4(8b_1 -$
$80b_2 + 560t)^2 - 3072a_2^2) - 675 - 10800(8b_1 - 80b_2 + 560t)^2 - 2764800a_2^2 + 691200(4a_1 - 24a_2)a_2 - 230400(4a_1 - 24a_2)^3 a_2)(2x -$
$12b_1 + 60b_2 - 280t)^2 + (-1680(4a_1 - 24a_2)^6(8b_1 - 80b_2 + 560t) - 460800(4a_1 - 24a_2)^3(8b_1 - 80b_2 + 560t)a_2 - 10800(4a_1 - 24a_2)^2(8b_1 -$
$272b_2 + 3248t) + 86400b_1 - 1209600b_2 + 10886400t + 43200(8b_1 - 80b_2 + 560t)^3 + 11059200(8b_1 - 80b_2 + 560t)a_2^2 + 3600(4a_1 -$
$24a_2)^4(8b_1 + 80b_2 - 1680t) - 86400(4a_1 - 24a_2)(16(8b_1 - 80b_2 + 560t)a_2 + 16a_2(32b_2 - 448t)))(2x - 12b_1 + 60b_2 - 280t) + 450(4a_1 -$
$24a_2)^4(-17 + 28(8b_1 - 80b_2 + 560t)^2 + 3072a_2^2) + 10800(4a_1 - 24a_2)(-16a_2 + 64(8b_1 - 80b_2 + 560t)^2 a_2 + 16384a_2^3) + 675(4a_1 -$
$24a_2)^2(-3 + 16(8b_1 - 80b_2 + 560t)^2 + 4096a_2^2 - 128(8b_1 - 80b_2 + 560t)(32b_2 - 448t)) + i(-345600(8b_1 - 80b_2 + 560t)^2 a_2 + (4a_1 -$
$24a_2)(2x - 12b_1 + 60b_2 - 280t)^{10} + (-60a_1 + 840a_2 + 5(4a_1 - 24a_2)^3)(2x - 12b_1 + 60b_2 - 280t)^8 + (-600a_1 - 240a_2 - 140(4a_1 -$
$24a_2)^3 + 10(4a_1 - 24a_2)^5 + 3840(4a_1 - 24a_2)^2 a_2)(2x - 12b_1 + 60b_2 - 280t)^6 + (-240(4a_1 - 24a_2)^3(8b_1 - 80b_2 + 560t) - 23040(8b_1 -$
$80b_2 + 560t)a_2 + 720(4a_1 - 24a_2)(8b_1 - 176b_2 + 1904t))(2x - 12b_1 + 60b_2 - 280t)^5 + 417600(4a_1 - 24a_2)^4 a_2 + (-450(4a_1 - 24a_2)^3 -$
$210(4a_1 - 24a_2)^5 + 10(4a_1 - 24a_2)^7 + 4800(4a_1 - 24a_2)^4 a_2 + 450(4a_1 - 24a_2)(-3 + 12(8b_1 - 80b_2 + 560t)^2 - 1024a_2^2) - 14400a_2 +$
$28800(4a_1 - 24a_2)^2 a_2)(2x - 12b_1 + 60b_2 - 280t)^4 + (-480(4a_1 - 24a_2)^5(8b_1 - 80b_2 + 560t) + 230400(4a_1 - 24a_2)^2(8b_1 - 80b_2 +$

$560t)a_2+7200(4a_1-24a_2)(8b_1-48b_2+112t)-2400(4a_1-24a_2)^3(16b_1-128b_2+672t)-230400(8b_1-80b_2+560t)a_2-230400a_2$
$(32b_2-448t))(2x-12b_1+60b_2-280t)^3+(1710(4a_1-24a_2)^5-60(4a_1-24a_2)^7+5(4a_1-24a_2)^9-900(4a_1-24a_2)^3(7+4(8b_1-$
$80b_2+560t)^2-3072{a_2}^2)+675(4a_1-24a_2)(7+16(8b_1-80b_2+560t)^2+4096{a_2}^2)-345600a_2-345600(8b_1-80b_2+560t)^2a_2-$
$88473600{a_2}^3-115200(4a_1-24a_2)^4a_2)(2x-12b_1+60b_2-280t)^2+(-240(4a_1-24a_2)^7(8b_1-80b_2+560t)-115200(4a_1-24a_2)^4(8b_1-$
$80b_2+560t)a_2+10800(4a_1-24a_2)(24b_1-400b_2+3920t+4(8b_1-80b_2+560t)^3+1024(8b_1-80b_2+560t){a_2}^2)+3600(4a_1-$
$24a_2)^3(24b_1-176b_2+784t)+720(4a_1-24a_2)^5(56b_1-400b_2+1680t)+345600(8b_1-80b_2+560t)a_2+691200a_2(32b_2-448t)-$
$43200(4a_1-24a_2)^2(16(8b_1-80b_2+560t)a_2+16a_2(32b_2-448t)))(2x-12b_1+60b_2-280t)+90(4a_1-24a_2)^5(-107+28(8b_1-$
$80b_2+560t)^2+3072{a_2}^2)+5400(4a_1-24a_2)^2(176a_2+64(8b_1-80b_2+560t)^2a_2+16384{a_2}^3)-225(4a_1-24a_2)^3(11+80(8b_1-$
$80b_2+560t)^2+20480{a_2}^2+128(8b_1-80b_2+560t)(32b_2-448t))-675(4a_1-24a_2)(-7+56(8b_1-80b_2+560t)^2+22528{a_2}^2-$
$128(8b_1-80b_2+560t)(32b_2-448t)-128(32b_2-448t)^2)-1440(4a_1-24a_2)^8a_2-9600(4a_1-24a_2)^6a_2-2764800(8b_1-80b_2+$
$560t)a_2(32b_2-448t)-870(4a_1-24a_2)^7+25(4a_1-24a_2)^9+(4a_1-24a_2)^{11}-151200a_2+265420800{a_2}^3)-11520(4a_1-24a_2)^7a_2-$
$195840(4a_1-24a_2)^5a_2+86400(32b_2-448t)^2+(2x-12b_1+60b_2-280t)^{10}+27000(8b_1-80b_2+560t)^2+2190(4a_1-24a_2)^6+$
$495(4a_1-24a_2)^8+11(4a_1-24a_2)^{10}+23500800{a_2}^2$
and
$d(x,t)=2024-777600(8b_1-80b_2+560t)(32b_2-448t)+120(8b_1-80b_2+560t)(2x-12b_1+60b_2-280t)^9+1440(32b_2-448t)(2x-$
$12b_1+60b_2-280t)^7-115200(4a_1-24a_2)^7a_2-518400(4a_1-24a_2)^5a_2+265420800(8b_1-80b_2+560t)^2{a_2}^2+(-120(4a_1-24a_2)^2+$
$5760(4a_1-24a_2)a_2+120)(2x-12b_1+60b_2-280t)^8+(480(4a_1-24a_2)^2-240(4a_1-24a_2)^4+15360(4a_1-24a_2)^3a_2+2320+$
$2160(8b_1-80b_2+560t)^2+1290240{a_2}^2-92160(4a_1-24a_2)a_2)(2x-12b_1+60b_2-280t)^6+(-720(4a_1-24a_2)^4(8b_1-80b_2+$
$560t)-276480(4a_1-24a_2)(8b_1-80b_2+560t)a_2+4320(4a_1-24a_2)^2(8b_1-176b_2+1904t)-51840b_1+103680b_2+2177280t)(2x-$
$12b_1+60b_2-280t)^5+(-1440(4a_1-24a_2)^4+11520(4a_1-24a_2)^5a_2+240(4a_1-24a_2)^2(56+135(8b_1-80b_2+560t)^2-11520{a_2}^2)+$
$518400(4a_1-24a_2)a_2+345600(4a_1-24a_2)^3a_2+3360+32400(8b_1-80b_2+560t)^2-13824000{a_2}^2+86400(8b_1-80b_2+560t)(32b_2-$
$448t))(2x-12b_1+60b_2-280t)^4+(-960(4a_1-24a_2)^6(8b_1-80b_2+560t)+921600(4a_1-24a_2)^3(8b_1-80b_2+560t)a_2-43200(4a_1-$
$24a_2)^2(24b_1-272b_2+2128t)-7200(4a_1-24a_2)^4(48b_1-448b_2+2912t)+345600b_1-5529600b_2+53222400t-86400(8b_1-$
$80b_2+560t)^3-22118400(8b_1-80b_2+560t){a_2}^2+172800(4a_1-24a_2)(16(8b_1-80b_2+560t)a_2-16a_2(32b_2-448t)))(2x-12b_1+$
$60b_2-280t)^3+(13440(4a_1-24a_2)^6+240(4a_1-24a_2)^8-240(4a_1-24a_2)^4(-326+45(8b_1-80b_2+560t)^2-34560{a_2}^2)+480(4a_1-$
$24a_2)^2(-76+135(8b_1-80b_2+560t)^2+311040{a_2}^2)-4147200(4a_1-24a_2)^3a_2-414720(4a_1-24a_2)^5a_2-64800(4a_1-24a_2)(-96a_2+$
$64(8b_1-80b_2+560t)^2a_2+16384{a_2}^3)+12144-97200(8b_1-80b_2+560t)^2+8294400{a_2}^2+518400(32b_2-448t)^2)(2x-12b_1+$
$60b_2-280t)^2+(-360(4a_1-24a_2)^8(8b_1-80b_2+560t)-276480(4a_1-24a_2)^5(8b_1-80b_2+560t)a_2-1440(4a_1-24a_2)^6(8b_1-$
$240b_2+2800t)+32400(4a_1-24a_2)^4(8b_1+112b_2-2128t)+64800(4a_1-24a_2)^2(-40b_1+752b_2-7728t+4(8b_1-80b_2+560t)^3+$
$1024(8b_1-80b_2+560t){a_2}^2)-777600(4a_1-24a_2)(16(8b_1-80b_2+560t)a_2+32a_2(32b_2-448t))-172800(4a_1-24a_2)^3(48(8b_1-$
$80b_2+560t)a_2+16a_2(32b_2-448t))-648000b_1+8553600b_2-74390400t+259200(8b_1-80b_2+560t)^3+331776000(8b_1-80b_2+$
$560t){a_2}^2-1036800(8b_1-80b_2+560t)^2(32b_2-448t)+265420800{a_2}^2(32b_2-448t))(2x-12b_1+60b_2-280t)+80(4a_1-24a_2)^6(191+$
$63(8b_1-80b_2+560t)^2+6912{a_2}^2)+21600(4a_1-24a_2)^3(-368a_2+64(8b_1-80b_2+560t)^2a_2+16384{a_2}^3)+240(4a_1-24a_2)^4(599+$
$135(8b_1-80b_2+560t)^2-57600{a_2}^2-360(8b_1-80b_2+560t)(32b_2-448t))-16200(4a_1-24a_2)(496a_2+1280(8b_1-80b_2+$
$560t)^2a_2+65536{a_2}^3+2048(8b_1-80b_2+560t)a_2(32b_2-448t))+24(4a_1-24a_2)^2(3881+12150(8b_1-80b_2+560t)^2+7257600{a_2}^2+$
$21600(8b_1-80b_2+560t)(32b_2-448t)+21600(32b_2-448t)^2)-1920(4a_1-24a_2)^9a_2+518400(32b_2-448t)^2+356400(8b_1-$
$80b_2+560t)^2+518400(8b_1-80b_2+560t)^4+3720(4a_1-24a_2)^8+120(4a_1-24a_2)^{10}+(1+(2x-12b_1+60b_2-280t)^2+(4a_1-$
$24a_2)^2)^6+33973862400{a_2}^4+223948800{a_2}^2$
is a solution to the $(NLS6)$ *equation* (1).

Proof: It is sufficient to check that the relation (1) is verified if we replace the expression of the solution given by (4).

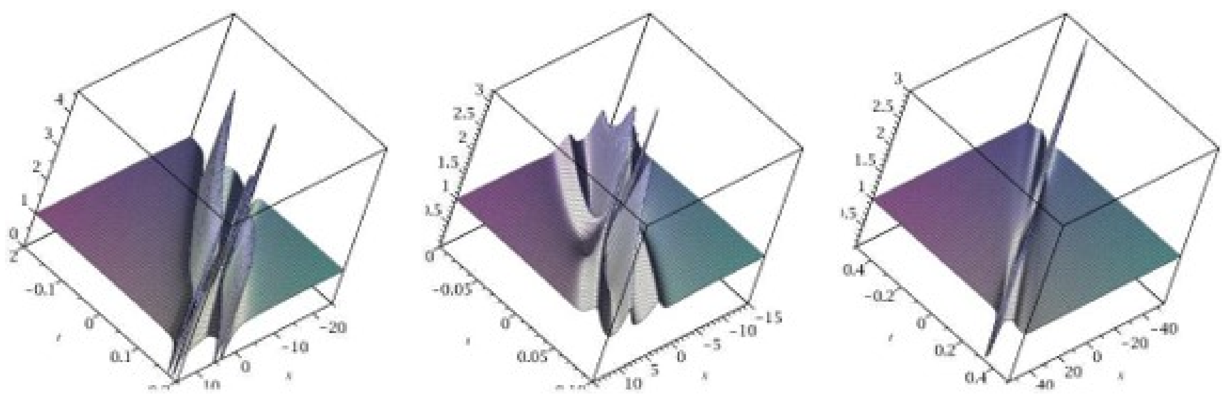

Figure 4: Solution of order 3 to (1); to the left $a_1=0, b_1=0, a_2=0, b_2=0$; in the center $a_1=1, b_1=0, a_2=0, b_2=0$; to the right $a_1=5, b_1=0, a_2=0, b_2=0$

We present the patterns of the modules of the solutions according to different values of the parameters.

We get also a non smooth solution of the equation (1).

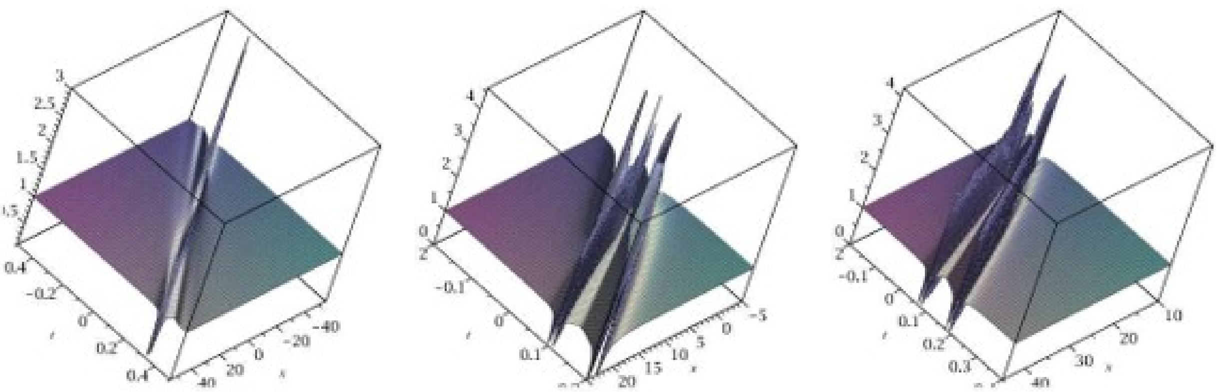

Figure 5: Solution of order 3 to (1); to the left $a_1 = 0, b_1 = 0,1, a_2 = 0, b_2 = 0$; in the center $a_1 = 0, b_1 = 1, a_2 = 0, b_2 = 0$; to the right $a_1 = 0, b_1 = 5, a_2 = 0, b_2 = 0$

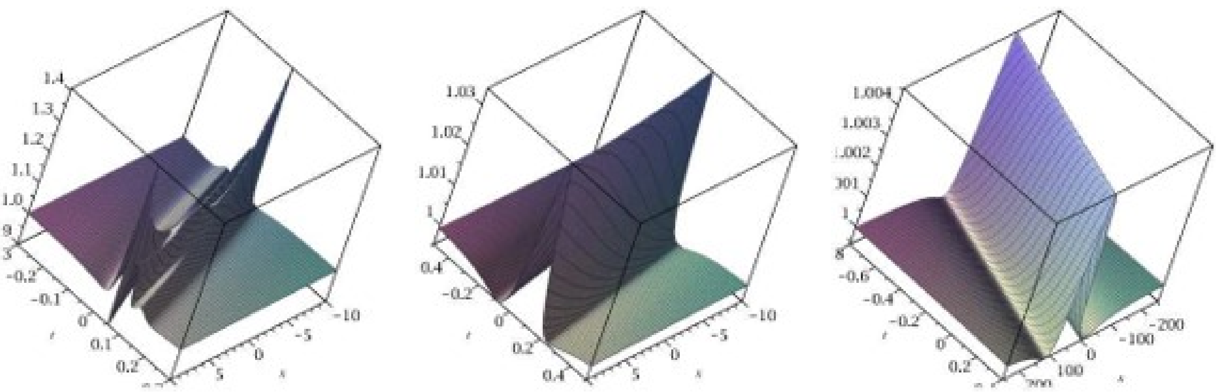

Figure 6: Solution of order 3 to (1); to the left $a_1 = 0, b_1 = 0, a_2 = 0,5, b_2 = 0$; in the center $a_1 = 0, b_1 = 0, a_2 = 1, b_2 = 0$; to the right $a_1 = 0, b_1 = 5, a_2 = 3, b_2 = 0$

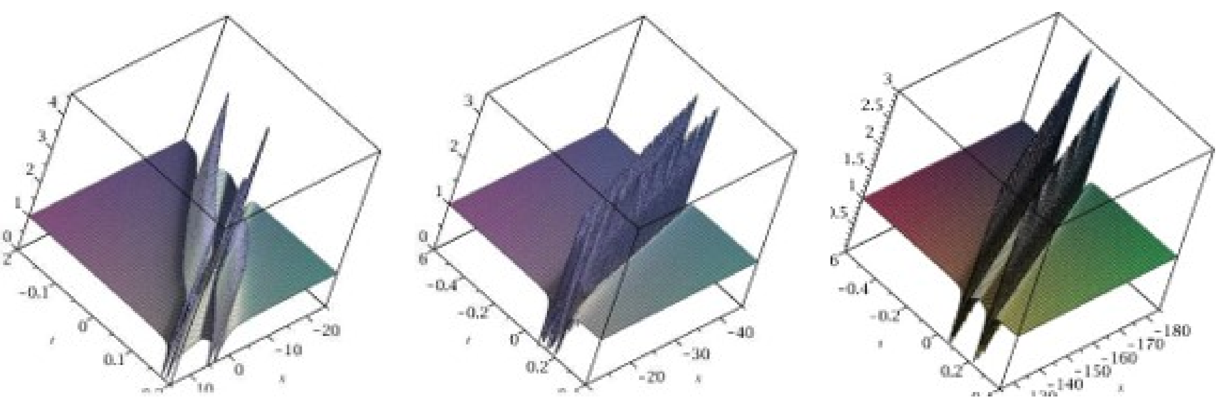

Figure 7: Solution of order 3 to (1); to the left $a_1 = 0, b_1 = 0, a_2 = 0, b_2 = 0,5$; in the center $a_1 = 0, b_1 = 0, a_2 = 0, b_2 = 1$; to the right $a_1 = 0, b_1 = 5, a_2 = 0, b_2 = 5$

5. Conclusion

For the first orders, rational solutions to the $(NLS6)$ equation have been constructed.
It is relevant to note that, we get a non smooth solution of the equation (1) as in the case of the mKdV equation [8] or the equation of order 4 of this NLS hierarchy [10].
The situation is contrary to odd-order equations, where we obtain rogue waves.
This is the case for the equation of order 1 (NLS), for the equation of order 3 (LPD) and for the equation of order 5. In these cases, according to the order N of the determinant giving the solutions of these equations, we obtain the following results.
Generically, for $N = 1$, there is a formation of a single peak, for $N = 2$, the formation of three peaks, and for $N = 3$, the formation of six peaks.
In all these N-order solutions we get quotient of a polynomial of degree $N(N+1)$ in x and t for the numerator by a polynomial of degree $N(N+1)$ in x and t for the denominator.
It is important to emphasize that unlike other equations belonging to this NLS hierarchy, as the NLS equation [7], the Lakshmanan Porsezian Daniel equation [11] or the eqution of order 5 of this hierarcht [12], we do not recover the structure of the triangles with the peaks that appear depending on the different values of the parameters.

Article Information

Disclaimer (Artificial Intelligence): The author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.), and text-to-image generators have been used during writing or editing of manuscripts.

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